# Physical model of current drive by ac helicity injection


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(Received 18 April 1984; accepted 9 May 1984)



It is shown that the ac helicity injection current drive described by Jensen and Chu and by Bevir and Gray can alternately be deduced from the pseudoelectric field $\sim\langle\widetilde{\mathbf{U}}\times\widetilde{\mathbf{B}}\rangle/c$ produced by oscillating velocity and magnetic field vectors $\widetilde{\mathbf{U}}$ and $\widetilde{\mathbf{B}}$ (here $\langle\ \rangle$ denote time average).


Bevir and Gray[1] proposed that steady-state plasma current drive could be achieved in a torus by adding a cos $\omega t$ component to the toroidal magnetic field and then oscillating the poloidal field $\sim\sin\omega t$. Their theory is a time-dependent version of Taylor's[2] theory of plasma relaxation (i.e., the plasma relaxes to a nearly force free equilibrium consistent with global conservation of helicity). Shoenburg *et al.*[3] reported recent preliminary tests of this idea on the Los Alamos ZT-40M theta pinch. Also recently, Jensen and Chu[4] presented explicit expressions for conservation of helicity and showed that Bevir and Gray's idea can be thought of as "ac helicity injection." Janos[5] has proposed using ac helicity injection to obtain steady-state operation in a spheromak. In this brief communication I would like to describe a simple physical model which produces the same result as the more formal approach of Jensen and Chu. Because of the intuitive nature of this model it is easy to see where the critical assumptions occur.

Jensen and Chu's helicity conservation equation is

$$\frac{\partial K}{\partial t}+\nabla\cdot\mathbf{Q}=-2\eta\mathbf{J}\cdot\mathbf{B}, \tag{1}$$

where the helicity is $K=\int\mathbf{A}\cdot\mathbf{B}\,dV$, the helicity flux is $\mathbf{Q}=2\Phi_{\text{elect}}\,\mathbf{B}+\mathbf{A}\times\partial\,\mathbf{A}/\partial t$, and $\eta$ is the resistivity. Jensen and Chu call the first term in $\mathbf{Q}$ the dc helicity flux (this exists when field lines intersect surfaces charged to finite potentials $\Phi_{\text{elect}}$), and they call the second term the ac helicity flux. It is easy to see that the quadrature poloidal-toroidal field phasing proposed by Bevir and Gray corresponds to generating an ac helicity flux, which can then balance the resistive loss term on the right-hand side of Eq. (1).

The purpose of this brief communication is to show that the ac helicity flux can be deduced directly from Ohm's law,

$$\mathbf{E}+\mathbf{U}\times\mathbf{B}/c=\eta\mathbf{J}. \tag{2}$$

In the typical situation where the plasma is stationary Eq. (2) reduces to $\mathbf{E}=\eta\mathbf{J}$ so that the resistive decay of the plasma current must be balanced by a dc electric field produced by induction, or by carefully located electrodes (dc helicity injection[4]). However, as Kadomtsev and Shafranov[6] have pointed out, one could envisage a situation where $\mathbf{E}=0$ and the resistive loss is balanced by the $\mathbf{U}\times\mathbf{B}$ term. Kadomtsev and Shafranov considered a steady outward plasma flow in the minor radius direction to provide the appropriate $\mathbf{U}$ (they suggested having a steady inward flux of neutrals which would ionize and then give the steady outward flow of plasma).

Now, suppose that instead of having a steady velocity there is (i) an oscillating velocity $\widetilde{\mathbf{U}}$ and also (ii) a magnetic field $\widetilde{\mathbf{B}}$ oscillating in phase with the velocity, so that in steady state

$$\langle\widetilde{\mathbf{U}}\times\widetilde{\mathbf{B}}\rangle/c=\eta\mathbf{J}, \tag{3}$$

where the brackets denote time average. Also suppose that the frequency $\omega$ of oscillation of $\widetilde{\mathbf{U}}$ and $\widetilde{\mathbf{B}}$ is much faster than the magnetic field resistive diffusion time $\tau_R$, but is much slower than the tearing time scale $\tau_T$ (i.e., $\tau_R^{-1}\ll\omega\ll\tau_T^{-1}$). Thus on the $\omega^{-1}$ time scale, the plasma acts as a perfect conductor but has an anomalously penetrated magnetic field (because of highly localized regions of tearing). Since the plasma is perfect conductor,

$$\widetilde{\mathbf{E}}+\widetilde{\mathbf{U}}\times\mathbf{B}/c=0, \tag{4}$$

or

$$\widetilde{\mathbf{U}}=c(\widetilde{\mathbf{E}}\times\mathbf{B})/B^2. \tag{5}$$

Substitution for $\widetilde{\mathbf{U}}$ in Eq. (3) gives

$$\mathbf{B}\langle\widetilde{\mathbf{E}}\cdot\widetilde{\mathbf{B}}\rangle-\langle\widetilde{\mathbf{E}}\widetilde{\mathbf{B}}\cdot\mathbf{B}\rangle=\eta\mathbf{J}B^2, \tag{6}$$

which dotted with $\mathbf{B}$ gives

$$\langle\widetilde{\mathbf{E}}\cdot\widetilde{\mathbf{B}}\rangle=\eta\mathbf{J}\cdot\mathbf{B}.$$

Assuming that $\widetilde{\mathbf{E}}$ is inductive this becomes

$$\left\langle-\frac{1}{c}\frac{\partial\widetilde{\mathbf{A}}}{\partial t}\cdot\nabla\times\widetilde{\mathbf{A}}\right\rangle=\eta\mathbf{J}\cdot\mathbf{B}. \tag{7}$$

Now, assume that $\widetilde{A}_{\rm pol} \sim \cos \omega t$ and $\widetilde{A}_{\rm tor} \sim \sin \omega t$ so that

$$\left\langle \frac{\partial \widetilde{\mathbf{A}}}{\partial t} \cdot \nabla \times \widetilde{\mathbf{A}} \right\rangle = \left\langle\left( \frac{\partial \widetilde{\mathbf{A}}_{\rm pol}}{\partial t} \cdot \nabla \times \widetilde{\mathbf{A}}_{\rm tor} + \frac{\partial \widetilde{\mathbf{A}}_{\rm tor}}{\partial t} \cdot \nabla \times \widetilde{\mathbf{A}}_{\rm pol} \right)\right\rangle$$
$$= - \left\langle \widetilde{\mathbf{A}} \cdot \nabla \times \frac{\partial \widetilde{\mathbf{A}}}{\partial t} \right\rangle$$

and Eq. (7) becomes

$$\frac{1}{2} \nabla \cdot \left\langle \frac{\partial \widetilde{\mathbf{A}}}{\partial t} \times \widetilde{\mathbf{A}} \right\rangle = \eta c \mathbf{J} \cdot \mathbf{B}, \tag{8}$$

which is just the ac component of the flux injection of Jensen and Chu [Eq. (1)]. Thus, ac helicity injection can be thought of as a mechanism for producing a "ponderomotive" effective dc electric field by nonlinear beating of appropriately phased velocity and magnetic field vectors.

Finally, if (following Taylor's theory) the equilibrium is nearly force free so that $\mathbf{J} \approx 4\pi c \mathbf{B}/a$ where $a \approx$ the minor radius, and all magnetic fields are expressed in terms of oscillating or static toroidal and poloidal fluxes, then the volume integral of Eq. (8) gives

$$\omega \tau_R \approx \frac{\Phi_{\rm tor}^2 + (a/2R)^2 \Phi_{\rm pol}^2}{\widetilde{\Phi}_{\rm pol} \widetilde{\Phi}_{\rm tor}}, \tag{9}$$

where $R$ is the major radius (also $\tau_R^{-1} \approx \eta c^2 R / \pi a^3$ has been assumed). To avoid adverse effects on particle confinement, one would want $\widetilde{\Phi}_{\rm pol} \ll \Phi_{\rm pol}$ and $\widetilde{\Phi}_{\rm tor} \ll \Phi_{\rm tor}$. From Eq. (9) this implies that $\omega \tau_R \gg 1$, consistent with the original assumption on $\omega$. In general, one would want to keep the magnitude of the oscillating fields as small as possible, by having $\omega$ as large as possible. Since $\tau_T^{-1}$ is the upper bound on $\omega$, the ratio between oscillating and static fluxes becomes

$$\frac{\widetilde{\Phi}_{\rm pol} \widetilde{\Phi}_{\rm tor}}{\Phi_{\rm tor}^2 + (a/2R)^2 \Phi_{\rm pol}^2} \geqslant \frac{\tau_T}{\tau_R}, \tag{10}$$

where the equality holds when $\omega \approx \tau_T^{-1}$. Hence it should be possible to have a steady-state current drive with small amplitude high-frequency quadrature phased toroidal and poloidal magnetic fields, providing the ratio of resistive to tearing time scales is large. If the tearing time follows the usual relation $\tau_T \sim \tau_R^{3/5} \tau_A^{2/5}$ (where $\tau_A$ is the Alfvén time), then $\tau_T / \tau_R \sim S^{-2/5}$ where $S = \tau_R / \tau_A$ is the magnetic Reynolds number.

It is interesting to calculate the magnitude and frequency of oscillations that would be required to sustain a tokamak fusion reactor with parameters: Deuterium plasma density $\approx 10^{14}$ cm$^{-3}$, $T \approx 10^4$ eV, $a \approx 1$ m, $R \approx 3$ m, $B \approx 5 \times 10^4$ G, and safety factor $q \approx 2$. For these parameters $\tau_R \approx 10^2$ sec, $\tau_A \approx 10^{-7}$ sec, $S \approx 10^9$, and $\Phi_{\rm tor} \approx \Phi_{\rm pol}$. Thus, ac helicity injection current drive would require oscillating fields with $\omega / 2\pi \approx 40$ Hz, and amplitudes about two percent of the steady-state magnitudes. These parameters should be technologically feasible.

## ACKNOWLEDGMENTS

The author wishes to thank Dr. T. H. Jensen, Dr. S. Zweben, and Dr. P. Liewer for several stimulating discussions.

This work was supported by National Science Foundation Grant No. ECS-8113533.

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